## ADVANCED GCE <br> MATHEMATICS

Core Mathematics 4

## QUESTION PAPER

Candidates answer on the printed answer book.
OCR supplied materials:

- Printed answer book 4724
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Thursday 16 June 2011
Afternoon
Duration: 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the printed answer book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.


## INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the question paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- The printed answer book consists of 16 pages. The question paper consists of 4 pages. Any blank pages are indicated.


## INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 Simplify $\frac{x^{4}-10 x^{2}+9}{\left(x^{2}-2 x-3\right)\left(x^{2}+8 x+15\right)}$.

2 Find the unit vector in the direction of $\left(\begin{array}{c}2 \\ -3 \\ \sqrt{12}\end{array}\right)$.

3 (i) Find the quotient when $3 x^{3}-x^{2}+10 x-3$ is divided by $x^{2}+3$, and show that the remainder is $x$.
(ii) Hence find the exact value of

$$
\begin{equation*}
\int_{0}^{1} \frac{3 x^{3}-x^{2}+10 x-3}{x^{2}+3} d x \tag{4}
\end{equation*}
$$

4 Use the substitution $x=\frac{1}{3} \sin \theta$ to find the exact value of

$$
\int_{0}^{\frac{1}{6}} \frac{1}{\left(1-9 x^{2}\right)^{\frac{3}{2}}} \mathrm{~d} x
$$

[6]

5 The lines $l_{1}$ and $l_{2}$ have equations

$$
\mathbf{r}=\left(\begin{array}{l}
4 \\
6 \\
4
\end{array}\right)+s\left(\begin{array}{l}
3 \\
2 \\
1
\end{array}\right) \quad \text { and } \quad \mathbf{r}=\left(\begin{array}{l}
1 \\
0 \\
0
\end{array}\right)+t\left(\begin{array}{r}
0 \\
1 \\
-1
\end{array}\right)
$$

respectively.
(i) Show that $l_{1}$ and $l_{2}$ are skew.
(ii) Find the acute angle between $l_{1}$ and $l_{2}$.
(iii) The point $A$ lies on $l_{1}$ and $O A$ is perpendicular to $l_{1}$, where $O$ is the origin. Find the position vector of $A$.

6 Find the coefficient of $x^{2}$ in the expansion in ascending powers of $x$ of

$$
\begin{equation*}
\sqrt{\frac{1+a x}{4-x}} \tag{8}
\end{equation*}
$$

giving your answer in terms of $a$.

7 The gradient of a curve at the point $(x, y)$, where $x>-2$, is given by

$$
\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{1}{3 y^{2}(x+2)}
$$

The points $(1,2)$ and $(q, 1.5)$ lie on the curve. Find the value of $q$, giving your answer correct to 3 significant figures.

8 A curve has parametric equations

$$
x=\frac{1}{t+1}, \quad y=t-1 .
$$

The line $y=3 x$ intersects the curve at two points.
(i) Show that the value of $t$ at one of these points is -2 and find the value of $t$ at the other point.
(ii) Find the equation of the normal to the curve at the point for which $t=-2$.
(iii) Find the value of $t$ at the point where this normal meets the curve again.
(iv) Find a cartesian equation of the curve, giving your answer in the form $y=\mathrm{f}(x)$.

9 (i) Show that $\frac{\mathrm{d}}{\mathrm{d} x}(x \ln x-x)=\ln x$.
(ii)


In the diagram, $C$ is the curve $y=\ln x$. The region $R$ is bounded by $C$, the $x$-axis and the line $x=\mathrm{e}$.
(a) Find the exact volume of the solid of revolution formed by rotating $R$ completely about the $x$-axis.
(b) The region $R$ is rotated completely about the $y$-axis. Explain why the volume of the solid of revolution formed is given by

$$
\begin{equation*}
\pi \mathrm{e}^{2}-\pi \int_{0}^{1} \mathrm{e}^{2 y} \mathrm{~d} y, \tag{4}
\end{equation*}
$$

and find this volume.

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